

# Baryonium, tetraquark state and glueball in large- $N_c$ QCD

Chun Liu<sup>a</sup>

Institute of Theoretical Physics, Chinese Academy of Sciences, P.O. Box 2735, Beijing 100080, P.R. China

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**Abstract.** From the large- $N_c$  QCD point of view, baryonia, tetraquark states, hybrids, and glueballs are studied. The existence of these states is argued for. They are constructed from baryons. In  $N_f = 1$  large- $N_c$  QCD, a baryonium is always identical to a glueball with  $N_c$  valence gluons. The ground state  $0^{-+}$  glueball has a mass of about 2450 MeV.  $f_0(1710)$  is identified as the lowest  $0^{++}$  glueball. The lowest four-quark nonet should be  $f_0(1370)$ ,  $a_0(1450)$ ,  $K_0^*(1430)$  and  $f_0(1500)$ . Combining with the heavy quark effective theory, spectra of heavy baryonia and heavy tetraquark states are predicted.  $1/N_c$  corrections are discussed.

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## 1 Introduction

The large- $N_c$  limit is one of the most important methods of non-perturbative QCD [1–3]. Properties of mesons can be observed from the analysis of planar diagrams and baryons from the Hartree–Fock picture. While mesons are free, the interaction between baryons is strong, scaling as  $\mathcal{O}(N_c)$ . This might imply a duality between the meson and baryon sectors [3]. This implication comes from the similarity between large- $N_c$  baryons and Polyakov–’t Hooft monopoles by taking  $1/N_c$  as a coupling constant. This duality was indeed partially realized [4] in the Skyrme model [5], in which a baryon is regarded as a soliton of a meson theory.

In this work, hadron spectra are described starting from baryons. Besides mesons and baryons, certain multi-quark systems are also included. Generally, color-singlet multi-quark–gluon systems have been expected naively due to their colorlessness. Now we try to give a deeper reasoning as regards their existence and their description from the large- $N_c$  QCD point of view. In principle, for large- $N_c$  QCD, baryon theories exist as ones dual to meson theories. Such theories, however, are strongly interacting, lacking a perturbative description. Taking baryons as the starting point can be traced back to Fermi and Yang [6] long before the establishment of QCD, while strange baryons were included by Sakita [7]. Its recent version can be found in [8]. Alternative to taking baryons as basic building blocks, we will still use the Hartree–Fock picture of large- $N_c$  baryons as Witten did [3] to do semi-quantitative analyses.

Baryons themselves have been studied in [3, 9–15]. There are  $N_c$  valence quarks in a baryon. Baryon–baryon

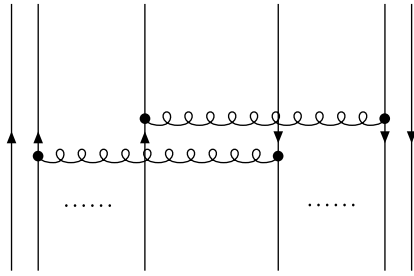
interactions are strong. Molecular states of baryons can exist due to their strong interactions. They are just the nuclei [3].

## 2 Baryonia

Let us consider baryon–antibaryon systems. The baryon–antibaryon system was mentioned in [3]. We will study its properties by assuming that it forms a bound state. The interaction between a baryon and an antibaryon can be as strong as that of baryon–baryon systems. Therefore, we expect that molecular states of a baryon and an antibaryon also exist. Because of baryon–antibaryon annihilation, the baryon and antibaryon in a baryonium is attractive at small distances, baryonia are more deeply bounded than nuclei.

The relevant interactions can be classified into two cases. The first one is that of glueball exchanges. The antibaryon inside a baryonium is not necessarily the antiparticle of the baryon in the baryonium. A possible example is the baryonium composed of  $\Delta^{++}(uuu)$  and  $\bar{\Delta}^-(\bar{d}\bar{d}\bar{d})$ , where the valence quark contents are given. In this case, the interaction inside a baryonium is described by Fig. 1 when the two baryons are close enough. Its  $N_c$ -dependence can be seen from the following. Each gluon–quark vertex contributes  $1/\sqrt{N_c}$ . There are  $N_c^2$  possible ways to make the first gluon. To make the second gluon, however, there are only  $N_c$  possible ways, because the two gluons must compose a color-singlet state. Therefore, the interaction energy is still proportional to  $N_c$ . Generally a baryonium mass is about  $2N_c\Lambda_{\text{QCD}}$ . The binding energy, though proportional to  $N_c$ , is expected to be smaller than the baryon masses  $\sim N_c\Lambda_{\text{QCD}}$ , because

<sup>a</sup> e-mail: liuc@mail.itp.ac.cn

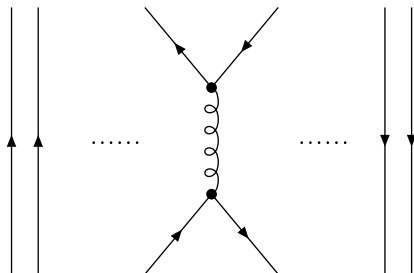


**Fig. 1.** Interaction inside a baryonium

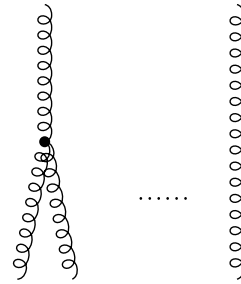
baryons are color singlets. This is consistent with the molecular picture of baryonia. In terms of the hadron language, the interaction is mediated by glueballs with the glueball–baryon coupling  $\sim \sqrt{N_c}$ . Because of heaviness of glueballs, such a  $t$ -channel glueball exchange interaction might be suppressed unless it happens at small distances. The short range interaction is also required by the confinement.

The second case is that of meson exchanges. When a quark and an antiquark have a common flavor in the baryonium, the interaction given by Fig. 2 [3] plays a role. This interaction is regarded as meson exchange. Once any of the quarks is able to annihilate with any of the antiquarks, namely, all the quarks have the same flavor in the baryonium, Fig. 2 is of equal  $N_c$  importance as Fig. 1. The realistic situation is an interplay of the two kinds of interactions described in Figs. 1 and 2. The amplitude of the baryon–antibaryon scattering described in 2 by assuming that  $N_f = 1$  [3] is proportional to  $N_c$ . This divergent large- $N_c$  behavior of the amplitude matches with that of baryon kinetic energies and shows the strong interacting behavior between the baryon and the antibaryon.

Assuming the existence of baryonia, an interesting observation on hybrids and glueballs can be seen in the large- $N_c$  limit. In the case that all the quarks have the same flavor in a baryonium ( $N_f = 1$ ), a hybrid state with valence content of  $(N_c - 1)$  quarks,  $(N_c - 1)$  antiquarks and one gluon, for instance, is large- $N_c$  enhanced due to the same reason of that the baryon interaction is strong, as can be seen by cutting Fig. 2 in the middle. This state and the baryonium constantly transfer into each other by the strong dynamics. Therefore in fact, this hybrid state is physically not different from the baryonium in



**Fig. 2.** Interaction inside a baryonium with a quark pair annihilation [3]



**Fig. 3.**  $N_c$  valence gluons transfer to  $N_c - 1$  valence gluons

the large- $N_c$  limit. For the same reason, this state can be equally identified as being composed of  $(N_c - 2)$  quarks,  $(N_c - 2)$  antiquarks and two gluons. Furthermore, and remarkably, a glueball composed of  $N_c$  valence gluons is also in fact indistinguishable from the baryonium. Referring to Fig. 2, it is easily seen that a valence gluon has a mass of 2 valence quarks, which is about  $2\Lambda_{\text{QCD}}$ . The glueball composed of two valence gluons therefore has a mass of about  $4\Lambda_{\text{QCD}}$ . It is clear that the glueball with  $N_c$  valence gluons has a mass of the baryonium. We have stated that the baryonium, the hybrid and the glueball are the same state in  $N_c = \infty$ ,  $N_f = 1$  QCD. Although at the quark–gluon level this is not true, there is no way to distinguish them at the hadron level due to the confinement.

On the other hand, the above reasoning also implies that the existence of glueballs supports the existence of baryonia. In the  $N_f = 0$  case, the confinement requires the existence of glueballs as hadrons. Light glueballs are massive with masses several times  $\Lambda_{\text{QCD}}$ . Adding a single flavor into this case, the lowest new hadrons include the  $\eta'$  meson due to spontaneous breaking of chiral symmetry and the anomaly, and the  $\Delta^{++}$  baryon. In the case of  $N_c \rightarrow \infty$ , glueballs and baryonia ( $\Delta^{++}$  and  $\bar{\Delta}^{--}$ ), with same quantum numbers, are indistinguishable. Baryonia are then generally expected. The constituent quark mass is determined to be half of the constituent gluon mass. With more flavor added, many new baryons with various flavor quantum numbers appear. The existence of baryonium beyond  $N_f = 1$  is less sound than the case of  $N_f = 1$ .

Note that a glueball of  $N_c$  valence gluons can make a transition to a glueball of  $(N_c - 1)$  valence gluons. But this transition rate is  $\mathcal{O}(1)$ . This is seen from Fig. 3. The three-gluon vertex has a factor of  $1/\sqrt{N_c}$ . There are  $N_c$  ways to make the transition. By considering the color quantum numbers of the gluons in Fig. 3, we know that the  $N_c$  ways do not add coherently. The rate of Fig. 3 is  $1/N_c$ . Then the total transition rate from a  $N_c$  valence gluon state into a  $(N_c - 1)$  valence gluon state is  $\mathcal{O}(1)$ . It is  $1/N_c$  suppressed compared to the transition rate of a baryonium into a one-gluon hybrid state. Therefore the  $N_c$  valence gluon state distinguishes itself from the  $(N_c - 1)$  valence gluon state. The two states have  $\mathcal{O}(1)$  mixing in the amplitudes. In other words, the number of valence gluons inside a glueball is well defined in the large- $N_c$  limit.

### 3 Diquark–antidiquarks

Unlike [3], we shall distinguish baryonium states and diquark–antidiquark states. Taking a color-singlet quark pair away from a baryonium, color-singlet  $(N_c - 1)$ -quark– $(N_c - 1)$ -antiquark states can always be formed. Their existence has been argued for in [3]. The  $(N_c - 1)$  quarks form a  $\bar{\mathbf{N}}_c$  representation, and  $(N_c - 1)$  antiquarks a  $\mathbf{N}_c$  representation. Such a state has a mass of about  $2(N_c - 1)\Lambda_{\text{QCD}}$ . It is a large- $N_c$  extension of the tetraquark state, and by taking  $N_c = 3$ , this state is the diquark–antidiquark one. In such a  $(N_c - 1)$ -quark– $(N_c - 1)$ -antiquark system, if all the quark flavors are the same, the hybrid state of  $(N_c - 2)$ -quarks,  $(N_c - 2)$ -antiquarks and one gluon is large- $N_c$  enhanced and is physically not different from the  $(N_c - 1)$ -quark– $(N_c - 1)$ -antiquark system.

Let us consider in more detail the  $(N_c - 1)$ -quark– $(N_c - 1)$ -antiquark state. Taking  $N_f = 1$ , processes similar to that described in Fig. 2 happen. In most of the cases when the gluon is formed from quarks with different colors, the interaction keeps the  $(N_c - 1)$  quarks in the  $\bar{\mathbf{N}}_c$  representation and  $(N_c - 1)$  antiquarks in the  $\mathbf{N}_c$  representation. This process amplitude is proportional to  $N_c$ . When the gluon is formed from quarks with the same color, the final  $(N_c - 1)$  quarks generally do not stay in  $\bar{\mathbf{N}}_c$  representation. But this transition amplitude is  $\mathcal{O}(1)$ . A similar result can be obtained if we consider  $t$ -channel gluon exchanges. Therefore, the quark configuration of a state with the  $(N_c - 1)$  quarks in a  $\bar{\mathbf{N}}_c$  representation and  $(N_c - 1)$  antiquarks in a  $\mathbf{N}_c$  representation makes sense in the large- $N_c$  limit. In the real situation ( $N_c = 3$ ), this is to say that the  $\mathbf{3} \otimes \bar{\mathbf{3}}$  tetraquark configuration does not mix with the  $\mathbf{6} \otimes \bar{\mathbf{6}}$  tetraquark configuration when  $N_c = 3$  is considered to be large.

Furthermore taking a color-singlet quark pair away, a  $(N_c - 2)$ -quark– $(N_c - 2)$ -antiquark state is formed with a mass of about  $2(N_c - 2)\Lambda_{\text{QCD}}$ . The  $(N_c - 2)$  quarks form a  $\bar{\mathbf{N}}_c \otimes \bar{\mathbf{N}}_c$  representation, and  $(N_c - 2)$  antiquarks a  $\mathbf{N}_c \otimes \mathbf{N}_c$  representation. The situation is more complicated.

The above procedure might continue. Finally, a valence quark–antiquark state is formed with a mass of about  $2\Lambda_{\text{QCD}}$ , which is just a meson. (Note that chiral symmetry breaking cannot be accounted for in this framework.)

In the same manner as we have discussed for the baryonium and the glueball in the last section, the single flavor  $(N_c - 1)$ -quark and  $(N_c - 1)$ -antiquark system is not distinguishable from the glueball composed of  $N_c - 1$  valence gluons. And the two-quark and two-antiquark system is indistinguishable from the glueball composed of two valence gluons.

Once more flavors are included, the above large- $N_c$  consideration becomes more complicated. As an example, the  $(N_c - 1)$  quarks stay in the lowest energy state if they take  $(N_c - 1)$  different flavors. In this case, the  $(N_c - 1)$ -quark– $(N_c - 1)$ -antiquark state transition to the hybrid state of  $(N_c - 2)$  quarks,  $(N_c - 2)$  antiquarks and one gluon is  $1/N_c$  suppressed compared to the single flavor case. They cannot be the same state in the large- $N_c$  limit, but they have  $\mathcal{O}(1)$  mixing in amplitudes.

### 4 Decays and binding energies

Decays of baryonia and  $(N_c - 1)$ -quark– $(N_c - 1)$ -antiquark states have been discussed in [3]. A baryonium decays into one meson and a  $(N_c - 1)$ -quark– $(N_c - 1)$ -antiquark state. A  $(N_c - 1)$ -quark– $(N_c - 1)$ -antiquark state decays into one meson and a  $(N_c - 2)$ -quark– $(N_c - 2)$ -antiquark state. Such cascade decays continue until the final 4-quark state decays into two mesons. These decays are slow.

These decay rates are  $\mathcal{O}(1)$ . This is easy to see from the fact that a color-singlet quark pair drops out of a baryonium or of a  $(N_c - 1)$ -quark– $(N_c - 1)$ -antiquark state with an amplitude of  $\mathcal{O}(1)$ .

Theoretically, baryon–antibaryon systems might be difficult to deal with, because the typical energy of such an interaction is proportional to  $N_c$  [3], which is the same  $N_c$ -dependence of the baryon masses  $\sim N_c\Lambda_{\text{QCD}}$ . However, as we have argued in Sect. 2, it is expected that the baryon–antibaryon interacting energy is smaller than  $N_c\Lambda_{\text{QCD}}$  due to the confinement. Furthermore, it is expected from large- $N_c$  QCD that the typical baryon–baryon interacting energy is also proportional to  $N_c$  and phenomenologically, nuclear physics shows that the typical binding energy of a baryon inside a nuclear is only about a few MeV. Therefore, we expect that the binding energy of a baryonium is actually a lot smaller than a baryon mass. It makes a molecular description (in the QCD sense) of baryonium states meaningful. Such a system in fact can be well described by non-relativistic quantum mechanics. Consequently, baryon spins decouple from the dynamics of baryonia.

To express the problem more clearly, in the large- $N_c$  limit, the baryon–baryon binding energy is  $N_c\lambda$ , and the baryon–antibaryon binding energy is  $N_c\lambda'$ . The confinement argument gives  $\lambda < \Lambda_{\text{QCD}}$  and  $\lambda' < \Lambda_{\text{QCD}}$ .

The slow transition of a baryonium to a  $(N_c - 1)$ -quark– $(N_c - 1)$ -antiquark state and one meson implies that the interacting strength inside the baryonium and the  $(N_c - 1)$ -quark– $(N_c - 1)$ -antiquark state are the same. In other words, in the large- $N_c$  limit the constituent quark mass ( $\Lambda_{\text{QCD}}$ ) can be taken to be the same in these two kinds of hadrons.

### 5 Analysis

We will make a numerical illustration through analyzing a realistic situation. In the semi-quantitative analysis, we only consider ground state hadrons. To be accurate requires the specification of the meaning of the  $\Lambda_{\text{QCD}}$  we have used. This quantity describes the energy of an individual quark inside baryons. In the following, we define  $\bar{\Lambda}_{\text{QCD}}$  to replace  $\Lambda_{\text{QCD}}$ ,

$$M_{\text{ground state baryon}} \equiv N_c \bar{\Lambda}_{\text{QCD}}, \quad (1)$$

namely  $\bar{\Lambda}_{\text{QCD}}$  is identified as a constituent quark mass in baryons. Taking  $N_c = 3$ , we have  $\bar{\Lambda}_{\text{QCD}} = (362 \pm 50)$  MeV by taking the average of masses of a nucleon and  $\Delta^{++}$ .

The analysis depends crucially on how much the baryonium binding energy is. A different binding energy corresponds to a different physical picture of hadrons. As we will see, it determines which group of hadron particles in the Particle Data Book is identified as 4-quark states. We will mainly take a 10 MeV binding energy. The error of this 10 MeV binding energy is hard to estimate. Some other phenomenological works [18–22] use about 300 MeV binding energy. A significantly larger binding energy will be considered briefly for comparison later.

The numerical analysis would be more appropriate to the  $N_f = 1$  case; however, it will go beyond that to  $N_f = 2$  and 3 without further mention of that the latter cases are more dependent on the assumptions.

### 5.1 10 MeV binding energy

We take the binding energy to be 10 MeV, which means that  $\lambda' \simeq \lambda$  or a little bit larger, considering both  $\lambda$  and  $\lambda'$  to be essentially determined by  $\Lambda_{\text{QCD}}$  and the confinement. Nuclear physics tells us that  $N_c\lambda$  is about a few MeV. We expect that  $N_c\lambda' \simeq 10$  MeV typically. A recent study of the skyrmeon model shows that the baryonium binding energy is indeed about 10 MeV [16, 17].

Consider the case of only one flavor, the lowest baryonium is  $s$ -wave ( $\Delta^{++}\bar{\Delta}^{--}$ ), with the quark being the up-quark, its mass is about  $2M_{\Delta^{++}} - 10$  MeV  $\simeq 2450$  MeV. As we have argued, this state can also be identified as a  $0^{-+}$  ground state of a three-gluon glueball in the large- $N_c$  limit. The inferred constituent gluon mass is consistent with those obtained via other methods [23]. The actual glueball mass may be a bit lower than the above value, because the actual flavor number is more than one. With one more light flavor being introduced, numerically it is estimated that the mass of the lowest proton–antiproton molecular state is about  $2M_N - 10$  MeV  $\simeq 1866$  MeV. This molecular state mixes with the  $0^{-+}$  glueball. In the large- $N_c$  limit, this mixing depends on the large- $N_c$  generalization of the nucleon, which we do not consider in this paper.

Experimentally, the BES collaboration has found two baryonium candidates,  $X(1860)$  [24] and  $X(1835)$  [25]. They were then theoretically studied [16, 17, 26–34]. Considering the uncertainties of the binding energy, these states are consistent with our expectation. Furthermore, the corresponding  $0^{-+}$  state exists due to the different baryon spin combination. Their approximate degeneracy is a result of baryon spin decoupling. As a check,  $p\bar{n}$ ,  $n\bar{p}$  and  $n\bar{n}$  states should have a degenerate mass with  $p\bar{p}$ , which is about 1835 MeV or 1860 MeV. In the three light flavor case, the lowest baryonium ( $p, \bar{A}$ ) or ( $n, \bar{A}$ ) is expected to have a mass of  $M_N + M_A - 10$  MeV  $\simeq 2045$  MeV and the baryonium ( $A, \bar{A}$ ) with a mass of  $2M_A - 10$  MeV  $\simeq 2220$  MeV. ( $p, \bar{A}$ ) is consistent with the experiment  $\sim 2075 \pm 13$  MeV [35].

A  $(N_c - 1)$ -quark– $(N_c - 1)$ -antiquark state has a mass of about  $2(N_c - 1)\bar{\Lambda}_{\text{QCD}}$ . From the argument of last section, it is reasonable to take the constituent quark mass in a tetraquark state to be the same as that in a baryonium. In the case of only one flavor, the 4-quark state is ( $uu\bar{u}\bar{u}$ ).

Its lowest mass is estimated to be  $(2M_{\Delta^{++}} - 10 \text{ MeV}) - 2\bar{\Lambda}_{\text{QCD}} \simeq 1740 \pm 100$  MeV. This state can be also regarded as a ( $u\bar{u}g$ ) hybrid or a  $0^{++}$  ground state two-gluon glueball. It should be identified as  $f_0(1710)$  in our scheme. Our estimation is consistent with lattice calculations [36–40].

With one more flavor included, the lowest mass can be written as

$$2(N_c - 1)\bar{\Lambda}_{\text{QCD}} \simeq [M_{X(1835)} \text{ or } M_{X(1860)}] - 2\bar{\Lambda}_{\text{QCD}} \\ \simeq (1110 - 1140) \pm 100 \text{ MeV}. \quad (2)$$

This is the 4-quark ground state ( $ud\bar{u}\bar{d}$ ) with both spin and isospin 0. It can be identified as  $f_0(1370)$ , which has a mass ranging from 1200 to 1500 MeV [41]. As we have noted, it mixes with the hybrid state ( $q\bar{q}g$ ), with  $q$  standing for the  $u$ - or  $d$ -quark.

As we have seen that the 4-quark state mass estimation has an uncertainty  $\sim 200$  MeV, the above numbers are of limited use in practice. Our point is that within the uncertainty, there should be 4-quark states, and they indeed have an experimental correspondence. After the strange quark is introduced, three kinds of lowest diquarks can be formed,  $ud$ ,  $us$  and  $ds$ . The 4-quark states form a nonet. They were studied by many authors [42–58]. In our work, their mass differences are expected to be determined by the strange-quark mass, which is not subject to the large uncertainty of the large- $N_c$  approximation. So the 4-quark ground states are naturally identified as  $f_0(1370)$ ,  $a_0(1450)$ ,  $K_0^*(1430)$ ,  $f_0(1500)$ ; more explicitly,  $f_0(1370)(ud\bar{u}\bar{d})$ ,  $K_0^*(1430)(ud\bar{u}\bar{s}, udd\bar{s}, us\bar{u}\bar{d}, ds\bar{u}\bar{d})$ ,  $a_0(1450)(us\bar{d}\bar{s}, ds\bar{u}\bar{s})$ ,  $a_0(1450)(s(n\bar{n})_+ \bar{s})$ ,  $f_0(1500)(s(n\bar{n})_+ \bar{s})$ , where  $(n\bar{n})_{\pm} \equiv (u\bar{u} \pm d\bar{d})/\sqrt{2}$ . The mixing among the  $f_0$  states is an  $\mathcal{O}(1)$  effect. Note that our 4-quark state identification is different from most of the previous studies [42–57] where it is  $\sigma$ ,  $\kappa$ ,  $a_0(980)$  and  $f_0(980)$  that are taken to be the lowest  $0^{++}$  4-quark states.

### 5.2 Large binding energy

For comparison, let us consider the case of large baryonium–antibaryon interacting energies. We know that  $\lambda \simeq \lambda'$  is still an assumption. If the annihilation effect is important,  $\lambda'$  is possibly a lot larger. References [18–22] took it to be about 200 MeV, which still makes the molecular picture of baryonia sensible. Now we fix  $N_c\lambda'$  by requiring that 4-quark ground states correspond to  $\sigma$ ,  $\kappa$ ,  $a_0(980)$  and  $f_0(980)$ . For a large binding energy, the mass of a diquark–antidiquark system is written as  $2(N_c - 1)\bar{\Lambda}'_{\text{QCD}}$ , while the mass of a baryonium is  $2N_c\bar{\Lambda}'_{\text{QCD}}$ . It is reasonable for ground states that  $\bar{\Lambda}'_{\text{QCD}} \simeq \bar{\Lambda}_{\text{QCD}}$  in the large- $N_c$  limit, because we can imagine a dynamical  $\mathcal{O}(1)$  process to generate a diquark–antidiquark in the ground state from a ground state baryonium via emitting a meson with the mass of  $2\bar{\Lambda}_{\text{QCD}}$ . The interaction energy between a diquark–antidiquark and a meson is  $1/N_c$  suppressed compared to  $N_c\lambda'$  and will be neglected, as we have also implicitly done in the last subsection. Taking the strange-quark mass  $m_s = 150$  MeV, from  $M_{f_0(980)} \simeq 2(N_c - 1)\bar{\Lambda}'_{\text{QCD}} + 2m_s$ , we obtain that  $\bar{\Lambda}'_{\text{QCD}} \simeq 170$  MeV. In this case,  $M_{\sigma} \simeq 680$  MeV.

The lowest  $0^{-+}$  baryonia should then be about 1020 MeV, which means a 960 MeV binding energy. However, there is no such baryonium correspondence in the Particle Data Book actually.

Therefore, considering a situation in practice, large- $N_c$  QCD analyses prefer a small baryonium binding energy. In the following we will not consider the large binding energy case.

### 5.3 Decay widths

A baryonium decays into one meson and one tetraquark state, and a tetraquark state into two mesons [3]. The decay rates are  $\mathcal{O}(1)$ . Therefore, from the large- $N_c$  point of view, diquark–antidiquark states decay slowly, and this also makes them distinguishable from two meson states. The  $0^{-+}$  baryonium  $X(1835)$  or  $X(1860)$  decays in  $p$ -wave into  $f_0(1370)$  and  $\sigma$ . Note that it cannot decay to  $s$ -wave  $f_0(1370)$  and  $\eta'$  due to the phase space. Therefore the dominant decay products are  $\rho\rho\pi\pi$ . This can be checked by future experiments.

## 6 Heavy hadrons

Now we consider the case of inclusion of a single heavy quark. Heavy quark effective theory (HQET) [59–61] provides a systematic way to investigate hadrons containing a single heavy quark. It is an effective field theory of QCD for such heavy hadrons. In the limit  $m_Q/\Lambda_{\text{QCD}} \rightarrow \infty$ , the heavy quark spin–flavor symmetry is explicit. The hadron mass is expanded as

$$M_H = m_Q + \bar{\Lambda}_H + \mathcal{O}(1/m_Q). \quad (3)$$

To obtain the HQET defined, universal heavy hadron mass  $\bar{\Lambda}_H$ , however, some non-perturbative QCD methods have to be used. We can apply the large- $N_c$  method. Heavy baryons were studied via this method [13–15, 62–72]. Let us first consider the relation of the quantity  $\bar{\Lambda}_H$  of a ground state heavy baryon and the nucleon mass. Heavy baryons contain  $(N_c - 1)$  light quarks, and one “massless” heavy quark (modular  $m_Q$ ). The mass or the energy of the baryon is determined by the summation of the energies of individual quarks. The kinetic energy of the heavy quark is typically  $\bar{\Lambda}_{\text{QCD}}$  like that of the light quark. The interaction energy between the heavy quark and any of the light quarks is typically  $\bar{\Lambda}_{\text{QCD}}/N_c$ . So the interaction energy between the heavy quark and the whole light quark system scales as  $\bar{\Lambda}_{\text{QCD}}$ . However, the total interaction energy of the light quark system itself scales as  $N_c \bar{\Lambda}_{\text{QCD}}$ . Therefore, in the large  $N_c$  limit,  $\bar{\Lambda}_H = M_N$ , where the uncertainty of the equation is  $\mathcal{O}(1) \sim \bar{\Lambda}_{\text{QCD}}$  in an  $1/N_c$  expansion.

Actually in the mass relation between heavy baryons and the corresponding light baryons in the large- $N_c$  limit, the uncertainty is smaller than  $\bar{\Lambda}_{\text{QCD}}$ . This is because the heavy quark constituent mass (modular  $m_Q$ ) does not deviate from  $\bar{\Lambda}_{\text{QCD}}$  very much. For an example, the  $\Lambda_Q$

baryon mass  $\bar{\Lambda}_{\Lambda_Q}$  is about 0.80 GeV [73]. It is more reasonable to take  $M_N - \bar{\Lambda}_{\Lambda_Q} \sim 0.15$  GeV to be the uncertainty in the following analysis.

Heavy baryonia containing a heavy quark are analyzed in the same large- $N_c$  spirit as in the light quark case. The  $0^-$  ground state baryonia  $(\Lambda_c, \bar{N})$  and  $(\Lambda_c, \bar{\Lambda})$  have masses

$$\begin{aligned} M_{(\Lambda_c, \bar{N})} &\simeq m_c - m_s + M_{(N, \bar{\Lambda})} \simeq 3.33 \pm 0.15 \text{ GeV}, \\ M_{(\Lambda_c, \bar{\Lambda})} &\simeq m_c - m_s + M_{(\Lambda, \bar{\Lambda})} \simeq 3.50 \pm 0.15 \text{ GeV}, \end{aligned} \quad (4)$$

where  $m_c$  is taken to be 1.43 GeV [73]. This is consistent with the naive estimation  $M_{(\Lambda_c, \bar{N})} = M_{\Lambda_c} + M_N - 10 \text{ MeV} \simeq 3.21 \text{ GeV}$  and  $M_{(\Lambda_c, \bar{\Lambda})} = M_{\Lambda_c} + M_{\Lambda} - 10 \text{ MeV} \simeq 3.40 \text{ GeV}$ . They also have corresponding degenerate  $1^-$  states due to baryon–spin decoupling.

For heavy diquarks, because of the heavy quark spin symmetry, the existence of a spin-zero diquark  $Qq$  implies that a  $Qq$  spin-one diquark also exists. This point was also noticed in [57]. The spectrum of the lowest 4-quark charm states is

$$\begin{aligned} M_{(cd, \bar{u}\bar{d})} &= M_{(cu, \bar{u}\bar{d})} \\ &= m_c + M_{f_0(1370)} \simeq (2.54\text{--}2.57) \pm 0.15 \text{ GeV}, \\ M_{(cd, \bar{u}\bar{s})} &= M_{(cu, \bar{u}\bar{s})} = M_{(cd, \bar{d}\bar{s})} = M_{(cu, \bar{d}\bar{s})} = M_{(cs, \bar{u}\bar{d})} \\ &= m_c + M_{a_0(1450)} \simeq 2.84 \pm 0.15 \text{ GeV}, \\ M_{(cs, \bar{u}\bar{s})} &= M_{(cs, \bar{d}\bar{s})} = m_c + M_{f_0(1500)} \simeq 2.94 \pm 0.15 \text{ GeV}. \end{aligned} \quad (5)$$

In the heavy quark limit, we have the degeneracy of the  $0^+$ ,  $1^+$  and  $2^+$  4-quark states.

Therefore, we expect a rich charm hadron spectrum ranging from 2.54 GeV to 3.50 GeV. The  $1/m_Q$  uncertainty is about  $\Lambda_{\text{QCD}}^2/m_c \sim 60 \text{ MeV}$ . The bottom case is the same except for a smaller  $1/m_Q$  correction because of the heavy quark flavor symmetry,

$$\begin{aligned} M_{(\Lambda_b, \bar{N})} &\simeq m_b - m_s + M_{(\Lambda, \bar{N})} \simeq 6.73 \pm 0.15 \text{ GeV}, \\ M_{(\Lambda_b, \bar{\Lambda})} &\simeq m_b - m_s + M_{(\Lambda, \bar{\Lambda})} \simeq 6.90 \pm 0.15 \text{ GeV}, \\ M_{(bd, \bar{u}\bar{d})} &= M_{(bu, \bar{u}\bar{d})} \\ &\simeq m_b + M_{f_0(1370)} \simeq (5.94\text{--}5.97) \pm 0.15 \text{ GeV}, \\ M_{(bd, \bar{u}\bar{s})} &= M_{(bu, \bar{u}\bar{s})} = M_{(bd, \bar{d}\bar{s})} = M_{(bu, \bar{d}\bar{s})} = M_{(bs, \bar{u}\bar{d})} \\ &\simeq m_b + M_{a_0(1450)} \simeq 6.24 \pm 0.15 \text{ GeV}, \\ M_{(bs, \bar{u}\bar{s})} &= M_{(bs, \bar{d}\bar{s})} \simeq m_b + M_{f_0(1500)} \simeq 6.34 \pm 0.15 \text{ GeV}, \end{aligned} \quad (6)$$

where  $m_b \simeq 4.83 \text{ GeV}$  [73].  $M_{(\Lambda_b, \bar{N})}$  and  $M_{(\Lambda_b, \bar{\Lambda})}$  are consistent with  $M_{\Lambda_b} + M_n - 10 \text{ MeV} \simeq 6.61 \text{ GeV}$  and  $M_{\Lambda_b} + M_{\Lambda} - 10 \text{ MeV} \simeq 6.80 \text{ GeV}$ , respectively.

For hadrons containing a pair of heavy quarks, the two heavy quarks intend to combine into a tighter object, which is described by non-relativistic QCD. However, if the two heavy quarks are separated by  $1/\Lambda_{\text{QCD}}$  or more in certain hadrons, the above HQET procedure can be applied to these hadrons. For examples, the following lowest baryonia

and 4-quark states may exist:

$$\begin{aligned}
M_{(\Lambda_c, \bar{\Lambda}_c)} &\simeq 2m_c - 2m_s + M_{(\Lambda, \bar{\Lambda})} \simeq 4.78 \pm 0.15 \text{ GeV}, \\
M_{(cu, \bar{c}\bar{u})} &= M_{(cu, \bar{c}\bar{d})} = M_{(cd, \bar{c}\bar{u})} = M_{(cd, \bar{c}\bar{d})} \\
&\simeq 2m_c + M_{f_0(1370)} \simeq (3.97-4.00) \pm 0.15 \text{ GeV}, \\
M_{(cu, \bar{c}\bar{s})} &= M_{(cd, \bar{c}\bar{s})} = M_{(cs, \bar{c}\bar{u})} = M_{(cs, \bar{c}\bar{d})} \\
&\simeq 2m_c + M_{a_0(1450)} \simeq 4.27 \pm 0.15 \text{ GeV}, \\
M_{(cs, \bar{c}\bar{s})} &\simeq 2m_c + M_{f_0(1500)} \simeq 4.37 \pm 0.15 \text{ GeV}, \quad (7)
\end{aligned}$$

where  $M_{(\Lambda_c, \bar{\Lambda}_c)}$  is consistent with  $2M_{\Lambda_c} - 10 \text{ MeV} \simeq 4.83 \text{ GeV}$ , and the uncertainty due to  $1/m_Q$  effects is  $\Lambda_{\text{QCD}}^2/2m_c \simeq 30 \text{ MeV}$ . The state  $(cs, \bar{c}\bar{s})$  is consistent with  $Y(4260)$  [56], which cannot be identified as  $(\Lambda_c, \bar{\Lambda}_c)$  [74] in our scheme. Considering the  $1/N_c$  uncertainties,  $X(3940)$  can be  $(cq\bar{c}\bar{q})$  ( $q = u, d$ ). In that case, a charged  $(cu\bar{c}\bar{d})$  state is also expected around 3940 MeV.

## 7 Summary and discussion

From the large- $N_c$  QCD point of view, we have considered baryonia, four-quark states, hybrids and glueballs. The existence of baryonium states is argued for from the existence of nuclei. These hadrons are constructed from baryons. We have argued that in  $N_f = 1$  large- $N_c$  QCD, a baryonium is always identical to a glueball with  $N_c$  valence gluons.  $f_0(1370)$ ,  $a_0(1450)$ ,  $K_0^*(1430)$  and  $f_0(1500)$  are identified as the lowest four-quark nonet. The glueball with three valence gluons has a mass of about 2450 MeV.  $f_0(1710)$  is identified as the glueball with two valence gluons. Combining with HQET, we have predicted heavy baryonia and heavy four-quark states.

This work can be viewed as a large- $N_c$  QCD extension of the Fermi–Yang model. A classification of hadrons is given a large  $N_c$  QCD basis. We have constructed hadron spectra from baryonia, because our starting point is baryons. The reversed procedure is not necessarily true. As an example, the one-gluon hybrid state existing in this scheme can always be generated from or identified as a diquark–antidiquark state. The large- $N_c$  QCD arguments and consequent estimation help us understanding the relevant experimental results. However, they do not result in a precise mathematical description for the hadrons we have studied in this paper. Finding such a systematic description is one of the tasks in solving nonperturbative QCD.

The uncertainties of the analysis should be discussed. Of course, the existence of baryonia as well as the 10 MeV binding energy is still an assumption, but it is supported by the large- $N_c$  analysis. Even if  $N_c = 3$ , baryonia are still expected. In many cases, the qualitative conclusion of large- $N_c$  QCD is also true when  $N_c = 3$ . The baryon–baryon strong interaction in large- $N_c$  QCD implies the existence of baryon bound states. Indeed in real QCD with  $N_c = 3$ , nuclei exist. The meson–meson interaction is vanishing in the large- $N_c$  limit; therefore, there is no molecular states of mesons. In the real world, molecular meson states seem non-existent.

It is important to discuss the  $1/N_c$  corrections to our numerical analysis. The estimated masses of the baryonia and diquark–antidiquark states would have  $\mathcal{O}(1) \sim (200-300) \text{ MeV}$  uncertainties. But the relative masses of the above hadrons do not have that large uncertainties. For example, once the binding energy of  $p\bar{p}$  is fixed as 10 MeV, then the  $\Lambda\bar{\Lambda}$  binding energy is 10 MeV with an uncertainty of about 30% due to SU(3) violation. Namely, the mass difference of the baryonia  $p\bar{p}$  and  $\Lambda\bar{\Lambda}$  is not subject to large  $1/N_c$  corrections. A more accurate treatment of baryonia can be similar to that in [9–15] by taking into account the baryonium binding energies. Furthermore, for the diquark–antidiquark states, their mass differences similarly have no large  $1/N_c$  uncertainties. As another example, the mass differences of the states  $(cd, \bar{u}\bar{d})$ ,  $(cd, \bar{u}\bar{s})$  and  $(cs, \bar{u}\bar{s})$  are not subject to the  $\Lambda_{\text{QCD}}$  uncertainty. The discovery of the baryonia and the diquark–antidiquark states with a single heavy quark and their mass estimation given in (4)–(6) will be tests of our understanding in the near future.

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